Chapter Ten.

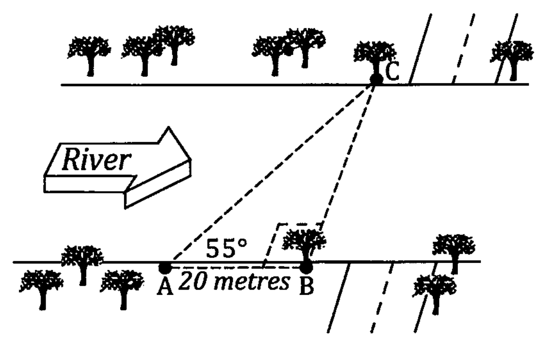
Trigonometry for right triangles.

Situation One

An emergency services team is called to an area that has experienced strong winds, torrential rains and some flooding. In one place a bridge has been washed away and needs to be replaced to maintain an essential supply route. The team contacts an army engineering unit for assistance. The unit can bring up and lay a ready made pontoon bridge provided they know the width of the river the bridge has to span.

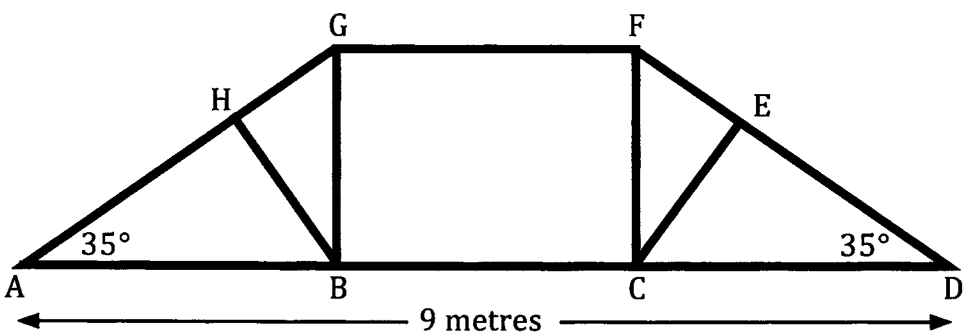
The emergency services team use a direction compass and tape measure to measure the angles and distance shown in the diagram (i.e. $\angle ABC = 90°$, $\angle CAB = 55°$ and AB = 20 metres).

Determine the width of the river.



Situation Two

A company manufacturing steel frameworks is asked to quote a price for the manufacture and delivery of fifty roof frames like the one shown sketched below.



In order to quote a price for the job the company needs to know, amongst other things, the total length of steel required to make each frame. Determine the total length of steel required for each frame, add 10% for joints and wastage and then round up to the next whole metre.

Note: AD is horizontal, BG and CF are vertical, AB = BC = CD and $\angle AHB = \angle DEC = 90°$.

How did you get on with the situations on the previous page?

Did you think of drawing scale diagrams to determine the required lengths?

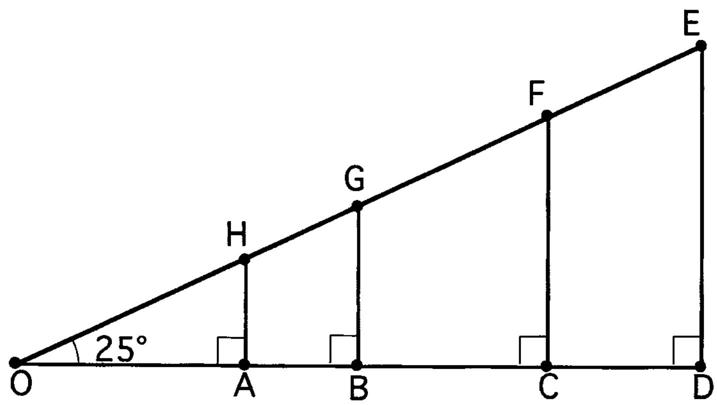
Perhaps instead you have encountered some *trigonometry* work in earlier years and remembered how to apply that to determine lengths of sides in right triangles.

Perhaps you used you calculator to determine lengths of unknown sides in right triangles.

In this chapter we will consider the use of *trigonometry* to determine sides and angles in right triangles.

Right angled triangles.

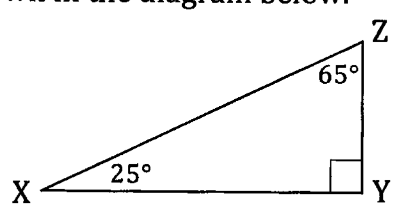
The four triangles OAH, OBG, OCF and ODE shown below all have angles of 25°, 65° and 90°. As you know from unit one of this course, the four triangles are **similar**.
Each triangle is an enlargement, or reduction, of the others.



In the above diagram measure the length of ED and the length of OD and use you calculator to determine: $\dfrac{\text{length of ED}}{\text{length of OD}}$.

Did you get an answer of approximately 0·47?

On a piece of A4 paper accurately draw a large triangle with angles of 25°, 65° and 90°.
Label your triangle XYZ as shown in the diagram below.



Measure the lengths of XY and YZ and determine: $\dfrac{\text{length of YZ}}{\text{length of XY}}$.

Did you again find that your answer was approximately 0·47 ?

Ask others in your class what value they got for $\dfrac{\text{length of YZ}}{\text{length of XY}}$.

Is everyone getting an answer of approximately 0·47?

Even though each person's triangle may be a little smaller or larger than another person's all triangles with angles of 25°, 65° and 90° are similar to each other and are like photographic enlargements or reductions of each other.

Again as we know from unit one of this course, if two sides in a triangle are in a certain ratio then the corresponding sides in any similar triangle will also be in the same ratio.

Thus for the diagram on the previous page, the ratio of any two of the sides in ΔOAH, e.g. $\dfrac{HA}{OA}$, will be the same as the ratio of the two corresponding sides in any of the other triangles,

$$\text{e.g.}\quad \frac{HA}{OA} = \frac{GB}{OB} = \frac{FC}{OC} = \frac{ED}{OD}$$

and will be equal to the corresponding ratio in any other triangle with angles of 25°, 65° and 90°.

$$\text{i.e.}\quad \frac{HA}{OA} = \frac{GB}{OB} = \frac{FC}{OC} = \frac{ED}{OD} = \frac{YZ}{XY} \approx 0{\cdot}47$$

Any triangle with angles of 25°, 65° and 90° will give this same answer when the length of the side "opposite the 25° angle" is divided by the length of the side "between the 25° angle and the right angle".

We call this ratio the **tangent** of 25°, abbreviated to **tan** 25°.

A more accurate value for tan 25° can be found from a calculator:

```
tan 25
                0.4663076582
```

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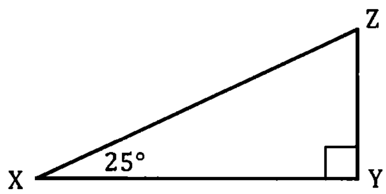
Trigonometry.

The **tangent** of an angle is one of three ratios commonly used in the branch of mathematics called **trigonometry**.

The three ratios are,

the **tangent** ratio (or **tan**), the **sine** ratio (or **sin**) and the **cosine** ratio (or **cos**).



$\tan 25° = \dfrac{ZY}{XY}$ $\sin 25° = \dfrac{ZY}{XZ}$ $\cos 25° = \dfrac{XY}{XZ}$

The values of these ratios can be obtained from a scientific, or graphic, calculator.

Correct to two decimal places:

$\tan 25° = 0{\cdot}47$

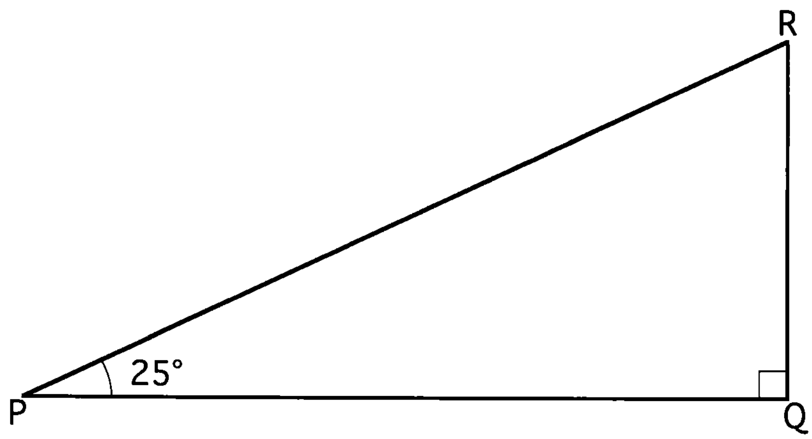
$\sin 25° = 0{\cdot}42$

$\cos 25° = 0{\cdot}91$

tan 25	0.4663076582
sin 25	0.4226182617
cos 25	0.906307787

Copy and complete the lines marked ✍ below for ΔPQR shown.

Measure the lengths as accurately as possible and perform the division using your calculator. Then see how the answers you obtain compare with the values given above.



✍ Side PQ is of length .... cm

✍ Side RQ is of length .... cm

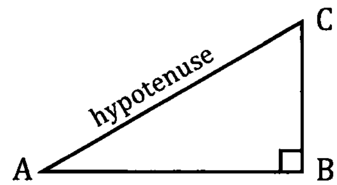
✍ Side PR is of length .... cm

✍ $\tan 25° = \dfrac{RQ}{PQ}$ = ...

✍ $\sin 25° = \dfrac{RQ}{PR}$ = ...

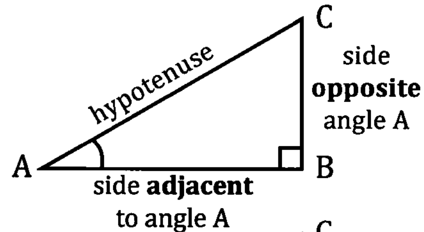
✍ $\cos 25° = \dfrac{PQ}{PR}$ = ...

Hypotenuse, opposite and adjacent.

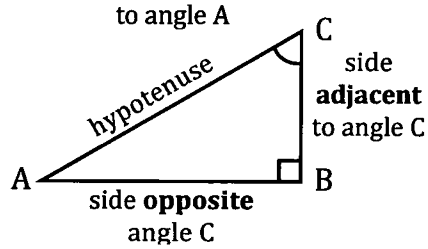


In a right triangle we call the side opposite the right angle the **hypotenuse**.

We then label the other two sides with respect to the angle we are considering:



With respect to angle A we say that CB is the **opposite** side and AB is the **adjacent** side.



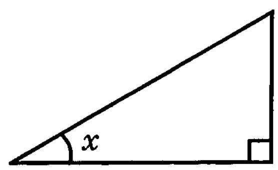
If, on the other hand, we are considering angle C it is now side AB that is the **opposite** side and BC is the **adjacent** side.

We then define the sine, cosine and tangent ratios as follows:

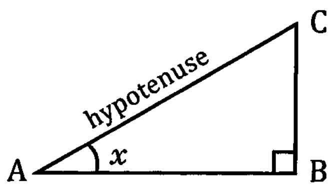
$$\sin x = \frac{\text{Opposite}}{\text{Hypotenuse}}$$

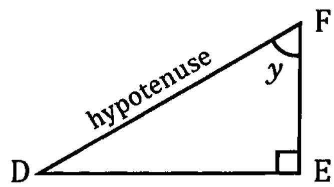
$$\cos x = \frac{\text{Adjacent}}{\text{Hypotenuse}}$$

$$\tan x = \frac{\text{Opposite}}{\text{Adjacent}}$$



Hence





$$\sin x = \frac{\text{Opposite}}{\text{Hypotenuse}} = \frac{CB}{AC} \qquad \sin y = \frac{\text{Opposite}}{\text{Hypotenuse}} = \frac{DE}{DF}$$

$$\cos x = \frac{\text{Adjacent}}{\text{Hypotenuse}} = \frac{AB}{AC} \qquad \cos y = \frac{\text{Adjacent}}{\text{Hypotenuse}} = \frac{EF}{DF}$$

$$\tan x = \frac{\text{Opposite}}{\text{Adjacent}} = \frac{CB}{AB} \qquad \tan y = \frac{\text{Opposite}}{\text{Adjacent}} = \frac{DE}{EF}$$

The sine, cosine and tangent ratios can be remembered using the mnemonic[1]

SOHCAHTOA

as shown on the next page.

[1] A mnemonic is a sequence of letters or words used to help us remember something.

SOHCAHTOA

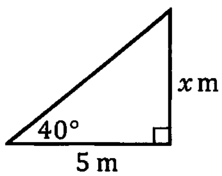
$$\text{Sin} = \frac{\textbf{Opposite}}{\textbf{Hypotenuse}} \qquad \text{Cos} = \frac{\text{Adjacent}}{\text{Hypotenuse}} \qquad \text{Tan} = \frac{\text{Opposite}}{\text{Adjacent}}$$

The following examples show how these ratios can be used to determine unknown sides and angles in right angled triangles.

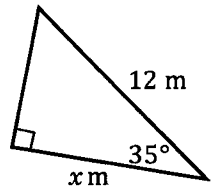
Example 1

Find the value of x in each of the following, correct to one decimal place.

(a)



(b)



(a) We require the side **opposite** the 40° angle and we know the **adjacent**. Thus we use the tangent ratio because it involves these two sides.

$$\tan 40° = \frac{\text{Opp}}{\text{Adj}}$$

$$\therefore \quad \tan 40° = \frac{x}{5}$$

Multiply both sides by 5 to eliminate fractions.

$$5 \times \tan 40° = x$$

we write this as

$$5 \tan 40° = x$$

Thus $\quad x = 4{\cdot}2$ (correct to 1 dp)

(b) We require the side **adjacent** to the 35° angle and we know the **hypotenuse.** Thus we use the cosine ratio because it involves these two sides.

$$\cos 35° = \frac{\text{Adj}}{\text{Hyp}}$$

$$\therefore \quad \cos 35° = \frac{x}{12}$$

Multiply both sides by 12 to eliminate fractions.

$$12 \times \cos 35° = x$$

we write this as

$$12 \cos 35° = x$$

Thus $\quad x = 9{\cdot}8$ (correct to 1 dp)

or, using the solve facility on a calculator:

$\text{solve}\left(\tan(40)=\frac{x}{5}, x\right)$

$\{x=4.195498156\}$

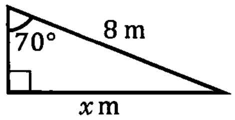
$\text{solve}\left(\cos(35)=\frac{x}{12}, x\right)$

$\{x=9.829824531\}$

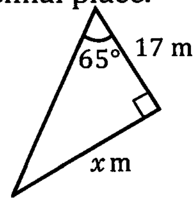
Example 2

Find the value of x in each of the following, correct to one decimal place.

(a)



(b)



(a) We require the side **opposite** the 70° angle and we know the **hypotenuse**. Thus we use the sine ratio.

$$\sin 70° = \frac{\text{Opp}}{\text{Hyp}}$$

$$\therefore \quad \sin 70° = \frac{x}{8}$$

i.e. $8 \sin 70° = x$

Thus $x = 7{\cdot}5$ (1 dp)

(b) We require the side **opposite** the 65° angle and we know the **adjacent**. Thus we use the tangent ratio.

$$\tan 65° = \frac{\text{Opp}}{\text{Adj}}$$

$$\therefore \quad \tan 65° = \frac{x}{17}$$

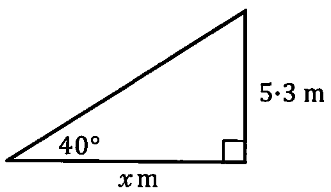
i.e. $17 \tan 65° = x$

Thus $x = 36{\cdot}5$ (1 dp)

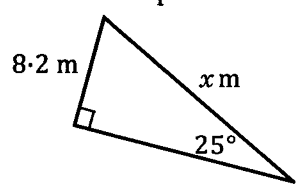
Example 3

Find the value of x in each of the following, correct to one decimal place.

(a)



(b)



(a) We require the side **adjacent** to the 40° angle and we know the **opposite** Thus we use the tangent ratio.

$$\tan 40° = \frac{\text{Opp}}{\text{Adj}}$$

$$\therefore \quad \tan 40° = \frac{5{\cdot}3}{x}$$

Multiply both sides by x to eliminate fractions.

$$x \tan 40° = 5{\cdot}3$$

Divide both sides by tan 40°.

$$x = \frac{5{\cdot}3}{\tan 40°}$$

Thus $x = 6{\cdot}3$ (1 dp)

(Or alternatively use the solve facility on a calculator to solve

$$\tan 40° = \frac{5{\cdot}3}{x} \text{)}$$

(b) We require the **hypotenuse** and we know the side **opposite** the 25° angle. Thus we use the sine ratio.

$$\sin 25° = \frac{\text{Opp}}{\text{Hyp}}$$

$$\therefore \quad \sin 25° = \frac{8{\cdot}2}{x}$$

Multiply both sides by x to eliminate fractions.

$$x \sin 25° = 8{\cdot}2$$

Divide both sides by sin 25°.

$$x = \frac{8{\cdot}2}{\sin 25°}$$

Thus $x = 19{\cdot}4$ (1 dp)

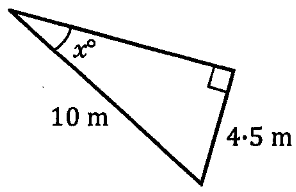
(Or alternatively use the solve facility on a calculator to solve

$$\sin 25° = \frac{8{\cdot}2}{x} \text{)}$$

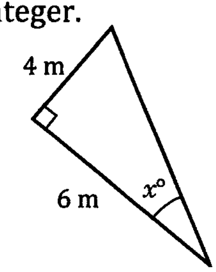
Example 4

Find the value of x in each of the following, to the nearest integer.

(a)



(b)



(a) We know the side **opposite** the required angle and we know the **hypotenuse**. Thus we use the sine ratio.

$$\sin x° = \frac{4{\cdot}5}{10}$$
$$= 0{\cdot}45$$

We require the angle whose sine is equal to 0·45.
Using "inverse sine" or "arc sine" on a calculator, often shown as $\sin^{-1}$, arcsin or perhaps ASIN we obtain $x = 27$ (nearest integer)

```
sin⁻¹(4.5÷10)
                  26.74368395
```

(b) We know the side **opposite** the required angle and the side **adjacent** to the required angle. Thus we use the tangent ratio.

$$\tan x° = \frac{4}{6}$$
$$= 0{\cdot}\bar{6}$$

We require the angle whose tangent is equal to $0{\cdot}\bar{6}$.
Using "inverse tan" or "arc tan" on a calculator, often shown as $\tan^{-1}$, arctan or perhaps ATAN we obtain $x = 34$ (nearest integer)

```
tan⁻¹(4÷6)
                  33.69006753
```

Notes regarding calculator usage.

1. We can use the solve facility on some graphic calculators to solve equations such as $\sin x° = \frac{4{\cdot}5}{10}$ and $\tan x° = \frac{4}{6}$. However, this needs care. In this chapter we are using sine, cosine and tangent in situations involving right triangles. We therefore know that when solving an equation like $\sin x = 0{\cdot}45$ our answer must be between 0° and 90°. However, in more advanced mathematics (some of which we will see in the next chapter), meaning can be given to the sine, cosine and tangent of angles that are bigger than 90° and even to the sine, cosine and tangent of negative angles! Whilst there is only one value of x between 0° and 90° for which $\sin x = 0{\cdot}45$ there are many angles outside this interval which have a sine equal to 0·45. The solve facility on a graphic calculator may give us one of these other answers instead. (See displays at the top of the next page.)

```
Eq: sin X=4.5÷10
  X = 153.256316
 Lft = 0.45
 Rgt = 0.45
```

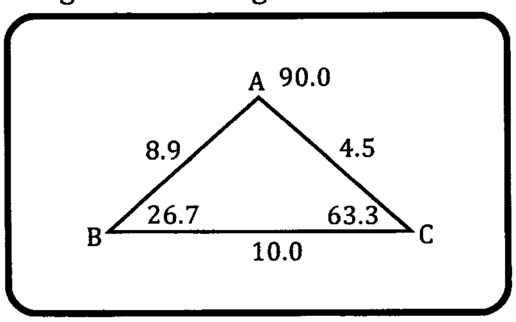
```
Eq: sin X=4.5÷10
  X = 746.743684
 Lft = 0.45
 Rgt = 0.45
```

However when an equation has more than one solution like this we can influence the one a calculator will give. On some calculators this is done by inputting an initial value of x and the calculator will tend to give the solution that is closest to this value. On other calculators we can instruct the calculator to only look for solutions in a particular range. In the display below left for example only solutions to the equation sin x = 0·45 in the interval 0° to 90° are asked for whereas below right solutions in the range 0° to 180° are requested.

```
solve(sin(x)=0·45, x)| 0≤x≤90
                {x=26.74368395}
```

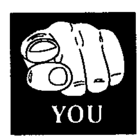
```
solve(sin(x)=0·45, x)| 0≤x≤180
 {x=153.256316, 26.74368395}
```

2. Some calculator programs allow the user to put in the known sides and angles of a triangle and, provided the information put in is sufficient, the program will determine the remaining sides and angles.



Other calculators can create a scale drawing of a geometrical figure and lengths and angles can then be determined from this drawing.
These programs can be useful but make sure that you understand the underlying ideas of sine, cosine and tangent and can reproduce the full method when required to do so.

Get to know your calculator.



Remember: When determining the lengths of sides in right triangles the **Pythagorean theorem** can also be of use in situations where we are given the lengths of two sides and need to find the length of the third, as was mentioned in the *Preliminary work* section at the beginning of this book.

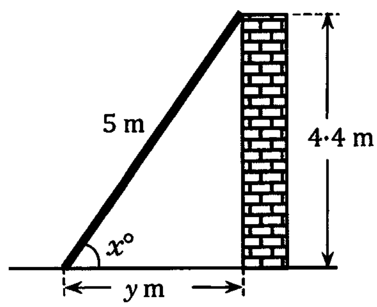
Applications.

The previous examples all involved abstract triangles in which we had to determine an unknown side length or angle size. Some questions will be more applied and will refer to a particular situation in which a right angled triangle is involved. A simple, neat, clear diagram will then need to be drawn.

Example 5

A ladder of length 5 metres leans against a vertical wall and just reaches the top of the wall. If the wall is 4·4 metres high calculate the angle the ladder makes with the horizontal ground (to the nearest degree) and the distance from the foot of the ladder to the wall (in metres correct to one decimal place).

First draw a diagram:



Knowing the hypotenuse and the side opposite the required angle we choose the sine ratio.

$$\sin x° = \frac{4{\cdot}4}{5}$$

Solving gives $x \approx 62$

Using Pythagoras' theorem.

$$5^2 = 4{\cdot}4^2 + y^2$$

Solving gives $y \approx 2{\cdot}375$

The ladder makes an angle of 62° with the horizontal ground and the foot of the ladder is 2·4 metres from the wall.

Notice

- The final answers are not given as $x = 62$ and $y = 2{\cdot}4$. The letters x and y were not part of the original question, we introduced them to help us obtain a solution. The final answer is given as a sentence that gives what was asked for.

- Having determined x, we could alternatively have then used "tan" or "cos" to determine the value of y:

$\tan x° = \frac{4{\cdot}4}{y}$	$\cos x° = \frac{y}{5}$
Multiply both sides by y to eliminate fractions.	Multiply both sides by 5 to eliminate fractions.
$y \tan x° = 4{\cdot}4$	$5 \cos x° = y$

Being sure to use the accurate value of x, not the rounded value of 62, solving gives:

$y \approx 2{\cdot}375$, as before.	$y \approx 2{\cdot}375$, as before.

Exercise 10A

1. Use your calculator to determine the following correct to 2 decimal places.

(a) sin 20°	(b) cos 10°	(c) tan 20°	(d) tan 40°
(e) tan 72°	(f) cos 53·4°	(g) sin 50°	(h) cos 40°

2. On a sheet of A4 paper accurately draw a large right triangle with angles of 35°, 55° and 90°. Measure the lengths of the three sides and **using these measurements**, and your calculator, determine estimates for

   sin 35° cos 35° tan 35° sin 55° cos 55° tan 55°

   and then check that your estimates are close to the accurate values the sin, cos and tan buttons on your calculator gives for these things.

3. Given that in each of the following x is one angle in a right triangle determine x in each case, giving your answer correct to one decimal place.

(a) $\sin x° = 0{\cdot}2$	(b) $\cos x° = 0{\cdot}4$	(c) $\tan x° = 1{\cdot}3$	(d) $\sin x° = 0{\cdot}3$
(e) $\cos x° = 0{\cdot}25$	(f) $\sin x° = 0{\cdot}8$	(g) $\tan x° = 2$	(h) $\cos x° = 0{\cdot}9$

4. Determine the value of x in each of the following, giving your answers correct to one decimal place.

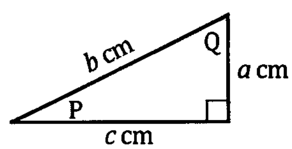
(a) $\sin 25° = \dfrac{x}{3}$	(b) $\cos 70° = \dfrac{x}{10}$	(c) $\tan 30° = \dfrac{x}{5}$
(d) $\sin 20° = \dfrac{7}{x}$	(e) $\cos 50° = \dfrac{9}{x}$	(f) $\tan 30° = \dfrac{7{\cdot}3}{x}$

5. Given that in each of the following x is one angle in a right triangle determine x in each case, giving your answer correct to one decimal place.

(a) $\sin x° = \dfrac{2}{5}$	(b) $\cos x° = \dfrac{5}{7}$	(c) $\tan x° = \dfrac{7}{5}$

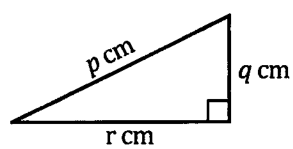
6. The right triangle shown on the right is scalene (i.e. the three sides of the triangle are of different lengths.) Write each of the following in terms of two of a, b and c.

(a) sin P	(b) cos P	(c) tan P
(d) cos Q	(e) sin Q	(f) tan Q



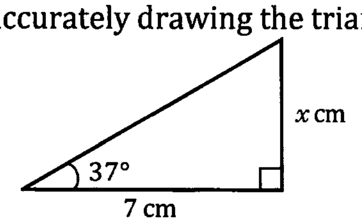
7. Which of the following statements are true for the right triangle shown on the right?

   $\boxed{q^2 = p^2 + r^2}$ $\boxed{p^2 = q^2 + r^2}$ $\boxed{r^2 = p^2 + q^2}$

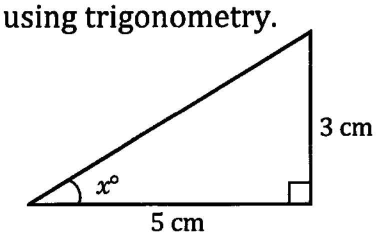


8. In each of the following determine x by:
   (i) accurately drawing the triangle, and (ii) using trigonometry.

(a)

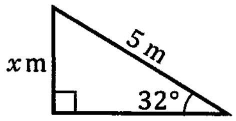


(b)

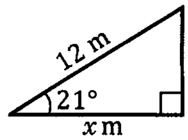


Find the value of x (and y if applicable) in each of numbers 9 to 30 clearly showing your use of trigonometry or Pythagoras in each one. (Give answers correct to one decimal place if rounding is necessary.)

9.



10.



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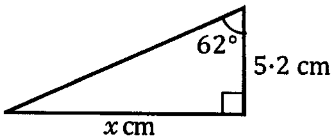

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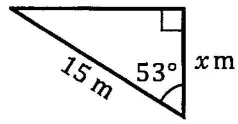

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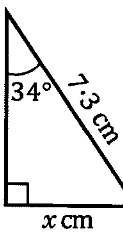

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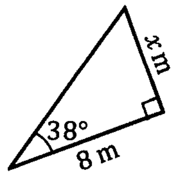

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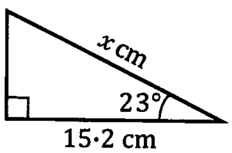

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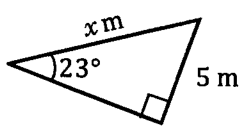

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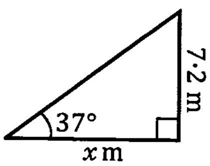

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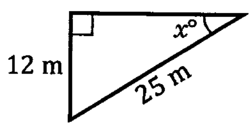

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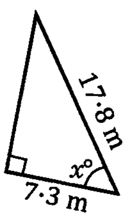

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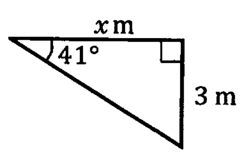

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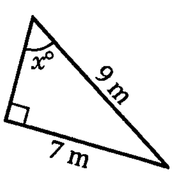

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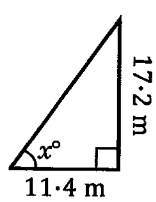

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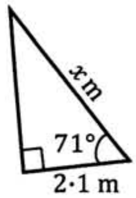

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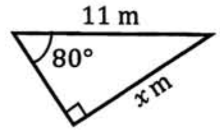

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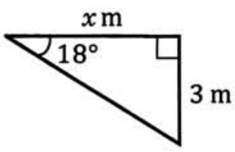

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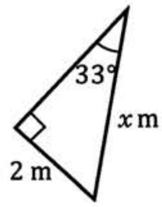

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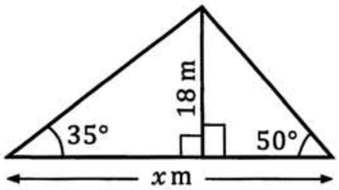

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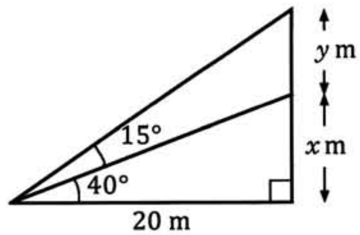

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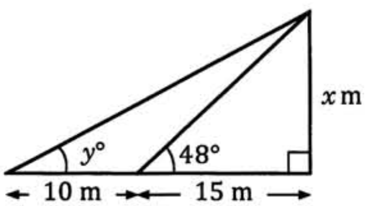

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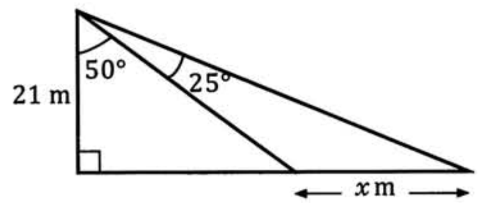
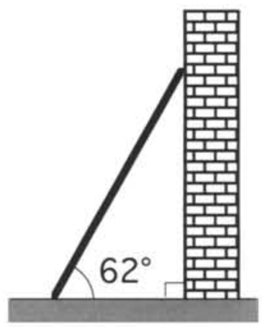


31. Triangle ABC is right angled at B. If AC = 17·6 cm and ∠CAB = 32° find
   (a) the length of AB, in centimetres correct to one decimal place,
   (b) the length of BC, to the nearest millimetre.

32. Triangle DEF is right angled at D. If ED = 7 cm and FD = 5 cm find
   (a) the size of ∠FED, to the nearest degree,
   (b) the length of FE, to the nearest millimetre.

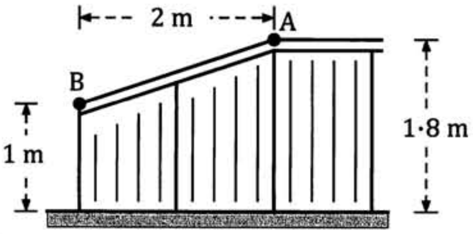
33. The diagram shows a ladder leaning against a vertical wall and making an angle of 62° with the horizontal ground.
   If the ladder is 8 metres in length calculate
   (a) how high the ladder reaches up the wall, to the nearest centimetre,
   (b) the horizontal distance from the foot of the ladder to the wall, to the nearest centimetre.



34. The outdoor light that illuminates the driveway of a two-storey house has a light globe that needs replacing. A ladder of length 5 metres is placed with its foot on the horizontal ground and 2 metres from the vertical wall of the house. In this position the ladder just reaches the light.
Find (a) the angle the ladder makes with the ground, to the nearest degree,
(b) the height of the light above the ground, in metres and correct to one decimal place.

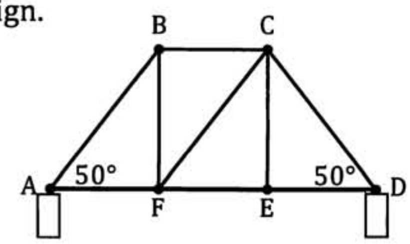
35. A person flying a kite holds the line 1 metre above level ground and has 45 metres of line out. If the line is straight and makes 62° with the horizontal what is the height of the kite above ground level (to the nearest metre)?

36. To reduce the force acting on the end of a garden fence due to the wind the fence can be "raked down".
The diagram on the right shows a fence raked down from a height of 1·8 metres to 1 metre in a horizontal distance of 2 m.

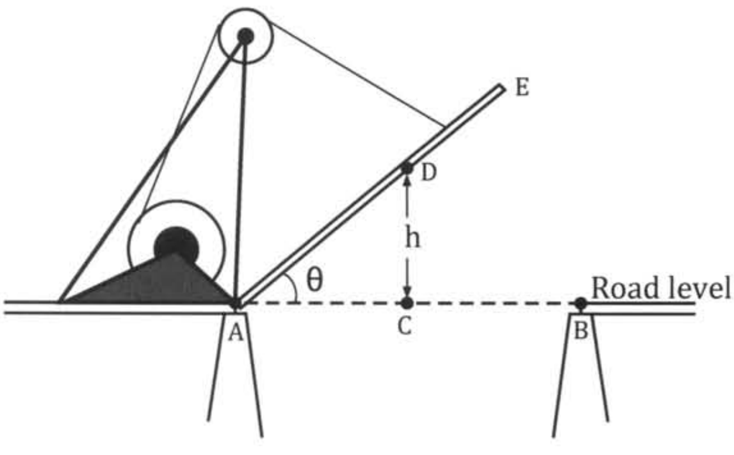


Find (a) the acute angle AB makes with the horizontal, to the nearest degree.
(b) the length of AB, to the nearest centimetre.

37. The diagram on the right shows a simple bridge design.
AD and BC are horizontal.
FB and EC are vertical.
$\angle BAF = \angle CDE = 50°$.
AD = 24 m and AF = FE = ED.
Calculate the lengths of AB, BF and FC giving all answers to the nearest centimetre.



38. The diagram shows a road bridge that can be opened to allow tall ships to pass underneath.
AB = AE = 20 metres and h is the distance from C, the mid-point of AB, to D, the point on the bridge vertically above C.
If h needs to be 8 m find θ in degrees correct to one decimal place.
For this value of θ find the length of AD as a percentage of AE (to the nearest percent).



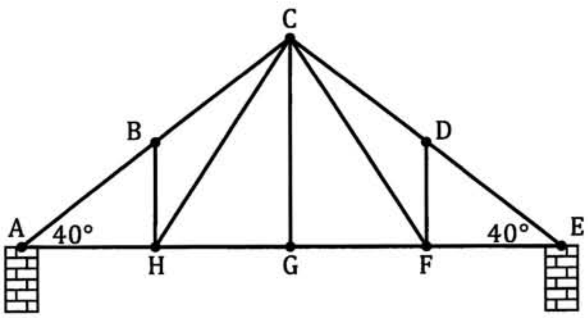
39. A vertical mast stands on level ground and is supported by a number of wires, as shown in the diagram. All these wires have one end attached to the ground, six metres from the base of the mast, and their other ends are attached to points that are either one third or two thirds of the way up the mast.

If the height of the mast is fifteen metres find:

(a) the length of one of the "short wires" (nearest centimetre) and the angle it makes with the ground (nearest degree),

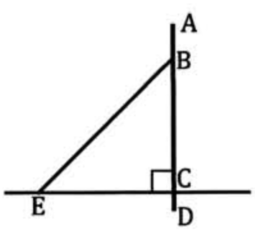
(b) the length of one of the "long wires" (nearest centimetre) and the angle it makes with the ground (nearest degree).

40. The diagram on the right shows the timbers forming part of a roof. The framework is symmetrical, AE is horizontal, HB, GC and FD are vertical and $\angle BAH = 40°$.
$AH = HG = GF = FE = 2$m.
Find the length of AC, CG, BH and HC giving your answers to the nearest centimetre.



41. A vertical pole of height 20 metres stands on horizontal ground and is supported by a number of guy wires. Each wire has one end attached to a point three-quarters of the way up the pole and the other end attached to one of the fastenings situated on the ground, 8 m from the base of the pole. Find the acute angle each wire makes with the horizontal, giving your answer to the nearest degree.

42. A mast AD is to stand vertically on horizontal ground. Part of the mast, CD in the diagram, is to be below ground. A straight support wire has one end fastened to the ground, at point E in the diagram, and the other to a point B on the mast. Angle BEC is to be no less than 42° and no more than 52° and BC will be no less than 9·9 metres and no more than 10·2 metres.

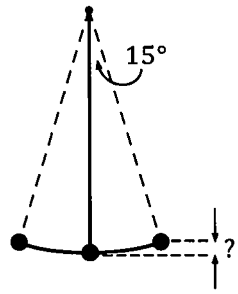


Based on these figures determine

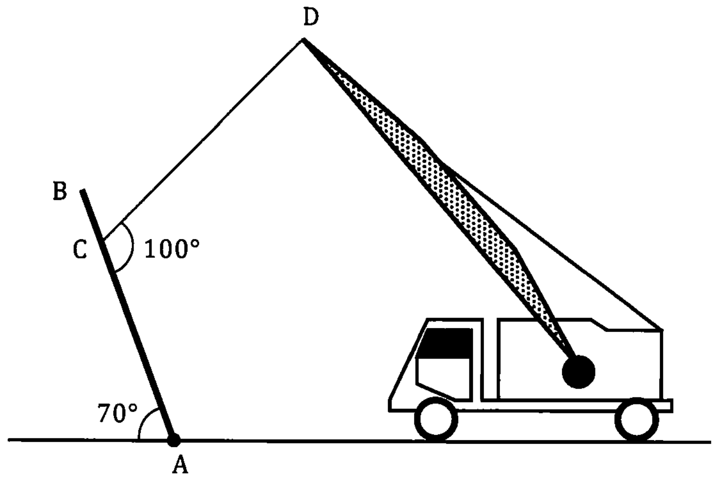
(a) the largest possible length of the support wire EB (nearest cm),

(b) the shortest possible length of the support wire EB (nearest cm).

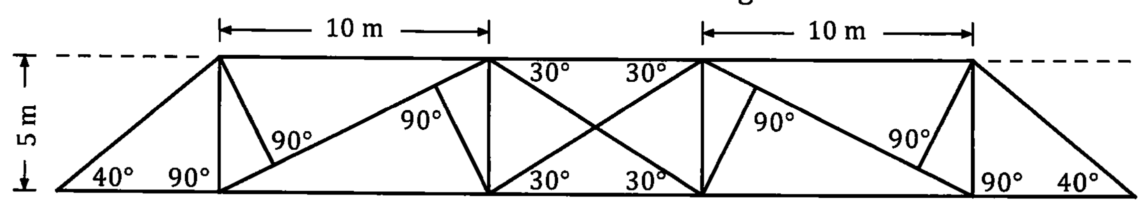
43. A pendulum of length 80 cm swings 15° either side of the vertical.
What is the vertical rise the bob of the pendulum makes above its lowest position, to the nearest millimetre?



44. The diagram shows a mobile crane lifting an 8 metre pole into the vertical position. The cable from the crane is attached to a point C where AC is four fifths of AB. At the instant shown in the diagram CD = 8 metres. How high is point D above the horizontal ground (to the nearest metre)?



45. (Challenging.)
The framework shown below is to be made out of lengths of steel.

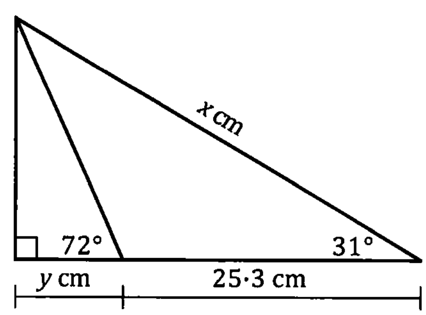


The framework consists of a right angled triangle on each end with three rectangles in the middle.
The company contracted to make it needs to know the total length of steel required.
Find the length of steel required, to the nearest whole metre.

46. (Challenging? Maybe, but hint makes it okay.)

Given the diagram shown on the right determine the values of x and y giving each answer correct to one decimal place.

Hint: Obtain the height in terms of y from one triangle, and then find the height in terms of y from another triangle and then ...



Accuracy and trigonometry questions.

So far all of the examples and almost all of the questions in this chapter have stated the degree of rounding that the answers should be given to. If this is not stated you should "round appropriately". Just what is appropriate depends upon the accuracy of the data we are given and what is appropriate for the situation. For example, if a question gives us a right triangle with one side of length 3·2 cm and one angle of size 36°, our calculator may give us the length of some other side as being 2·32493609 but it would be quite inappropriate to claim this sort of accuracy because it is far beyond the accuracy of the information used to obtain it.

In general, if we are not told the accuracy to give an answer to, our final answer should not be more accurate than the accuracy of the data we use to obtain it. If a question gives us a length in cm, to 1 decimal place, we should not claim greater accuracy for any lengths we determine. Sometimes we may need to use our judgement of the likely accuracy of the given data. Given a length of 5 cm we might assume this has been measured to the nearest mm and hence give answers similarly to the nearest mm. (Theoretically a measurement of 5 cm measured to the nearest mm should be recorded as 5·0 cm but this is often not done.)

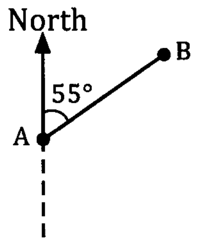
In situations where accuracy is crucial any given measurements could be given with "margins of error" included, for example 3·2 cm ± 0·05 cm, 36° ± 0·5°. More detailed error analysis could then be carried out and the margins of error for the answer calculated. However this is beyond the scope of this text and, as mentioned earlier: *If a question does not state the degree of rounding required your final answers should be rounded "appropriately".*

You are already used to rounding appropriately in some situations not involving trigonometry. For example if asked for the sale price in an "8% off everything sale" for something usually costing \$25·45 you would give the answer as \$23·40, not the \$23·414 value a calculator gives for \$25·45 × 0·92. If asked how many chocolate bars costing \$1·40 each we could purchase with \$8 we would not give the calculator answer of 5·714285714 bars, even though we would know that the values of \$1·40 and \$8 were exact. Instead we would say that 5 bars could be purchased and, if we wanted to give more information, we could add that \$1 change would be given.

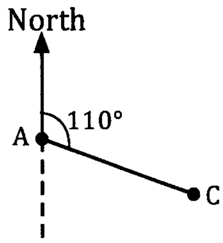
Some questions requiring the use of trigonometry involve situations that mention **bearings** and/or **angles of elevation or depression**. It is often these concepts that cause errors more so than the trigonometry itself. The next few pages cover these concepts

Bearings.

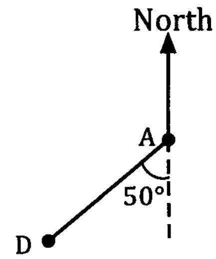
From one location, the direction we would need to travel to reach a second location can be given as a bearing. These are angles, expressed as three figures, and are measured from North, clockwise, as shown on the next page.



Bearing of B from A is
055°



Bearing of C from A is
110°



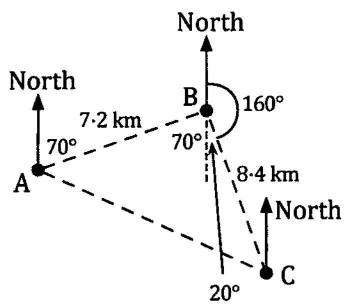
Bearing of D from A is
230°

Alternatively bearings can be expressed as a certain number of degrees (between 0 and 90) to the East or West of North or South. Thus the above *three figure bearings* could be expressed as the following *compass bearings*:

Bearing of B from A is	Bearing of C from A is	Bearing of D from A is
N 55° E	S 70° E	S 50° W

Example 6

From town A, town B lies 7·2 km away on a bearing of 070°.
From town B, town C lies 8·4 km away on a bearing of 160°.
Find the distance and bearing of C from A.



First make a sketch of the situation:
Note that with the given bearings $\angle ABC = 90°$.

In ΔABC, by Pythagoras' theorem $\quad AC^2 = 7{\cdot}2^2 + 8{\cdot}4^2$

Thus $\quad AC \approx 11{\cdot}06$ km

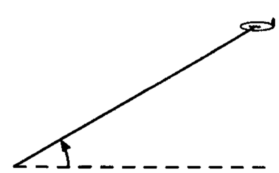
Also $\quad \tan \angle BAC = \dfrac{8{\cdot}4}{7{\cdot}2}$

giving $\quad \angle BAC \approx 49{\cdot}4°$

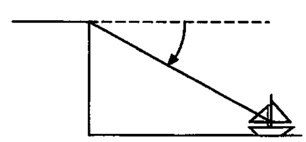
Thus the bearing of C from A is approximately 119°, i.e. (49° + 70°).
Town C is 11·1 km from A, on a bearing of 119°.

Elevation and Depression.

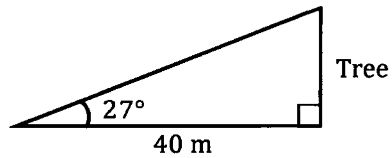
- **Angles of elevation** are measured from the horizontal, up.



- **Angles of depression** are measured from the horizontal, down.



Example 7
From a point on level ground, 40 metres from a tree, the angle of elevation of the top of the tree is 27°. Calculate the height of the tree.



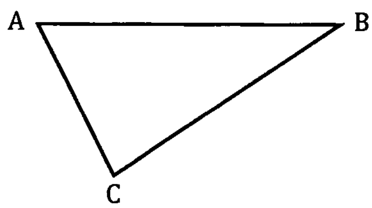
First make a sketch of the situation:
With respect to the 27° we know the length of the adjacent side and require the length of the opposite side. Thus we choose the tangent ratio.

$$\tan 27° = \frac{\text{height of tree}}{40}$$

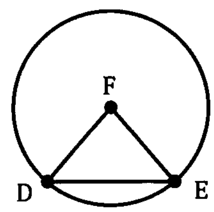
Solving gives: height of tree $\approx$ 20·4 metres
The height of the tree is approximately 20 metres.

More vocabulary.

- Note also that if a question refers to a line **subtending** an angle at a point this is the angle formed by joining each end of the line to the point.



Line AB subtends
∠ACB at C.

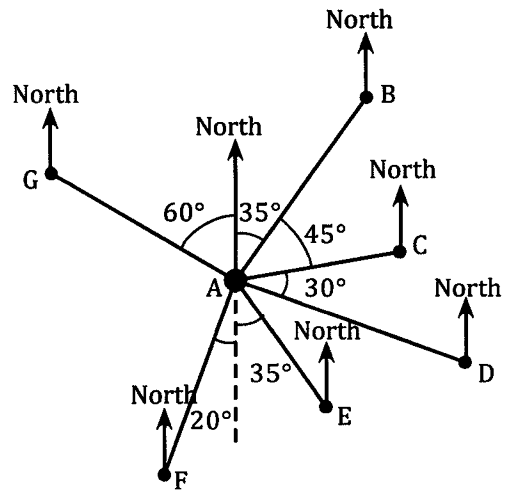


Chord DE subtends
∠DFE at the centre, F.

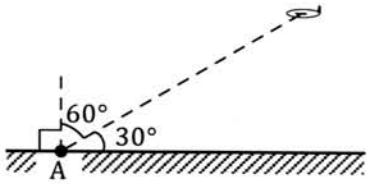
- If points are referred to as being **collinear** this means they lie in a straight line.

Exercise 10B

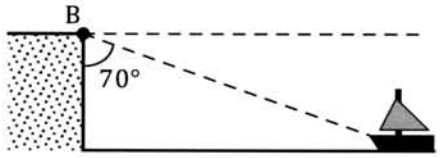
1. From the diagram on the right find:
   (a) the bearing of B from A,
   (b) the bearing of C from A,
   (c) the bearing of D from A,
   (d) the bearing of E from A,
   (e) the bearing of F from A,
   (f) the bearing of G from A,
   (g) the bearing of A from B,
   (h) the bearing of A from C,
   (i) the bearing of A from D,
   (j) the bearing of A from E,
   (k) the bearing of A from F,
   (l) the bearing of A from G.



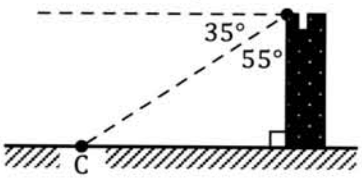
2. (a) What is the angle of elevation of the aeroplane from A?



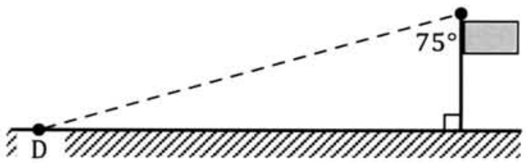
(b) What is the angle of depression of the ship from B?



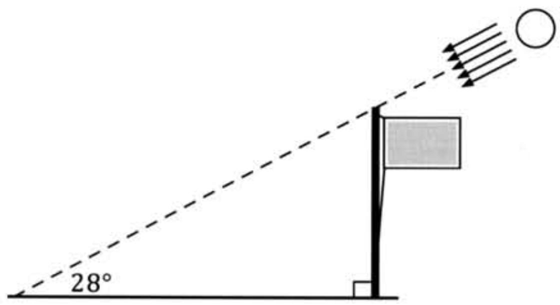
(c) What is the angle of depression of point C from the top of the tower?



(d) What is the angle of elevation of the top of the flagpole from D?



3. When the angle of elevation of the sun is 28° a vertical flag pole casts a shadow of length 22·4 metres on horizontal ground. Calculate the height of the flagpole.



4. Find the angle of elevation of the sun if a 2·0 metre pole held vertically on horizontal ground casts a shadow of length 4·1 metres. Give your answer correct to the nearest degree.

5. A flagpole stands vertically on level ground. When the sun's elevation is 24° the flagpole casts a shadow of length 22·5 metres. Find the height of the flagpole.

6. A and B are two points on horizontal ground. A mast of length 540 cm is to stand vertically with its base at B. From A, the top of the mast will have an angle of elevation of 17°. A straight wire is to run from the top of the mast to the point A. How far is this, rounded up to the next metre?

7. At 9 a.m. one morning two ships leave a harbour and head out to sea. One ship travels at a steady 4 km/h on a bearing 110° and the other ship maintains 5 km/h on a bearing 200°. To the nearest kilometre how far apart are the ships one and a half hours later?

8. The three points A, B and C lie on horizontal ground and form a straight line with B between A and C. A vertical tower of height 40 metres stands at C. The angle of elevation of the top of the tower is 18° from A and 35° from B. How far is B from A (to the nearest metre)?

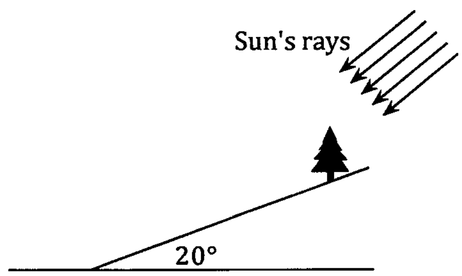
9. From ship A, ship B lies 12·2 km away on a bearing N58°W.
From ship B, ship C lies on a bearing S32°W.
If the bearing of C from A is S59°W how far is ship C from ship A?

10. Three collinear points A, B and C lie on horizontal ground with B between A and C. A vertical tower of height 42 metres stands at B. The angle of elevation of the top of the tower is 28° from A and 17° from C. How far is C from A (to the nearest metre)?

11. Three collinear points A, B and C lie on horizontal ground with B between A and C. A vertical tower of height 36 metres stands at C. The angle of elevation of the top of the tower is 15° from A and 40° from B. How far is B from A (to the nearest metre)?

12. Two vertical towers stand on level ground. From the top of one tower, of height 40 metres, the top and base of the second tower have angles of elevation and depression of 20° and 30° respectively. Find the height of the second tower.

13. A tree stands vertically on a hillside that is inclined at 20° to the horizontal. When the angle of elevation of the sun is 39° (i.e. 39° with the horizontal) the tree casts a shadow of length 35·3 metres straight down the slope. How tall is the tree?



14. A forest warden on fire look-out duty in an observation tower notices smoke directly North of his position. From a second tower, situated 5·3 km due East of the first, another warden sees the smoke on a bearing 335°. How far is the smoke from the first observation tower?

15. An observer in an aircraft flying at an altitude of 500 metres notices two ships at sea. At the moment the observer sees the ships as being "in line" he records their angles of depression as 30° and 40° respectively. How far apart are the ships?

16. A vertical flagpole stands on top of a vertical tower of height 40 m. At a point level with the base of the tower and 60 m from it, the flagpole subtends an angle of 10°. How long is the flagpole?

17. A and B are two points on level ground, 19·6 metres apart. A vertical flagpole at B subtends an angle of 40° at the eye of a person standing at A and whose "eye height" is 1·6 m. Find the height of the flagpole.

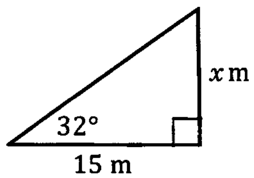
18. From a point on level ground the angle of elevation of a vertical flagpole is 40°. From the same position find the angle of elevation of the point three quarters of the way up the flagpole.

Miscellaneous Exercise Ten.

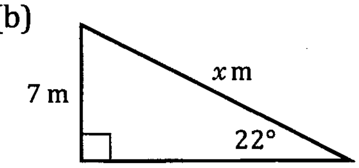
This miscellaneous exercise may include questions involving the work of this chapter, the work of any previous chapters, and the ideas mentioned in the preliminary work section at the beginning of the book.

1. Determine the value of x in each of the following, giving your answers correct to one decimal place.

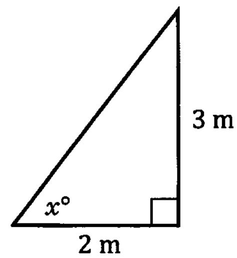
(a)



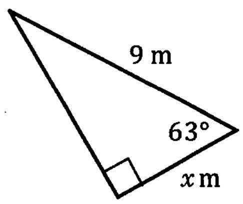
(b)



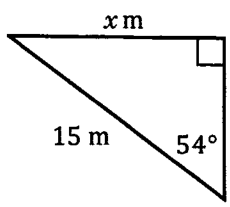
(c)



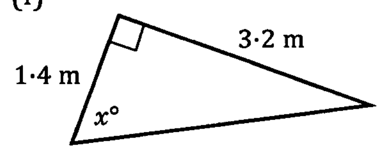
(d)



(e)



(f)



2. Find the mean and the median of the following six amounts:

$13 600 $5 700 $23 400 $2 100 $14 600 $98 700

3. The 35 students in a class sat a test that was marked out of 40. The 20 boys had a mean score of 24·35 and the class mean was 23. What was the mean score of the girls in the class?

4. Solve the following equations.

(a) $7x - 15 = 132$ (b) $3(2x - 1) + 2x = 17$

(c) $\dfrac{2x+1}{3} = 5$ (d) $\dfrac{5}{x} = 8$

5. Formula: $\boxed{v^2 = u^2 + 2as}$

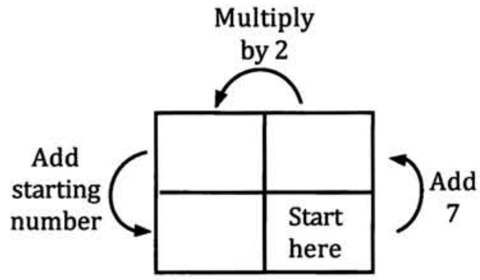
(a) Find s given that $v = 13$, $u = 5$ and $a = 24$.

(b) Find a given that $v = 21$, $u = 17$ and $s = 19$.

6. I think of a number, multiply it by three, add seven and then divide the answer by two. At the end of all this the number I end up with is eleven more than the number I first thought of. Find the number first thought of.

7. John takes out a loan which involves simple interest charged at the rate of 7·5% per annum. After 4 years John repays $11 180 which clears the loan and interest. How much did John borrow in the first place?

8. The "rectapatterns" shown below all follow the pattern shown on the right.
Copy and complete the eight "rectapatterns" shown below.



(a)

	5

(b)

	7·5

(c)

	92

(d)

	17

(e)

36	

(f)

14	

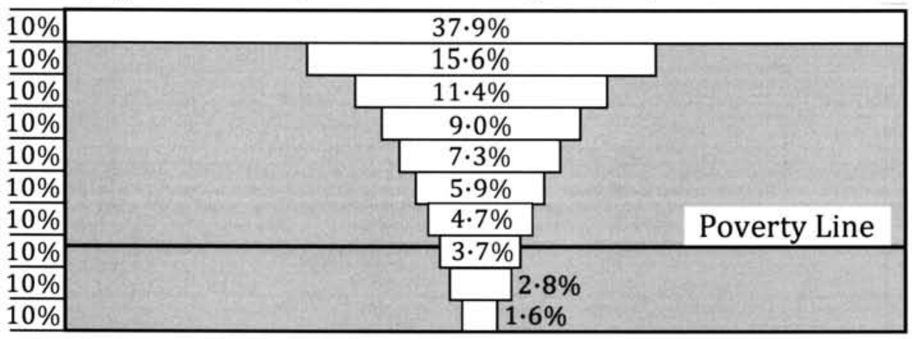
(g)

59	

(h)

83	

9. The diagram below shows the percent of total national income earned by each tenth of the population of a particular country in one year, richest at the top.



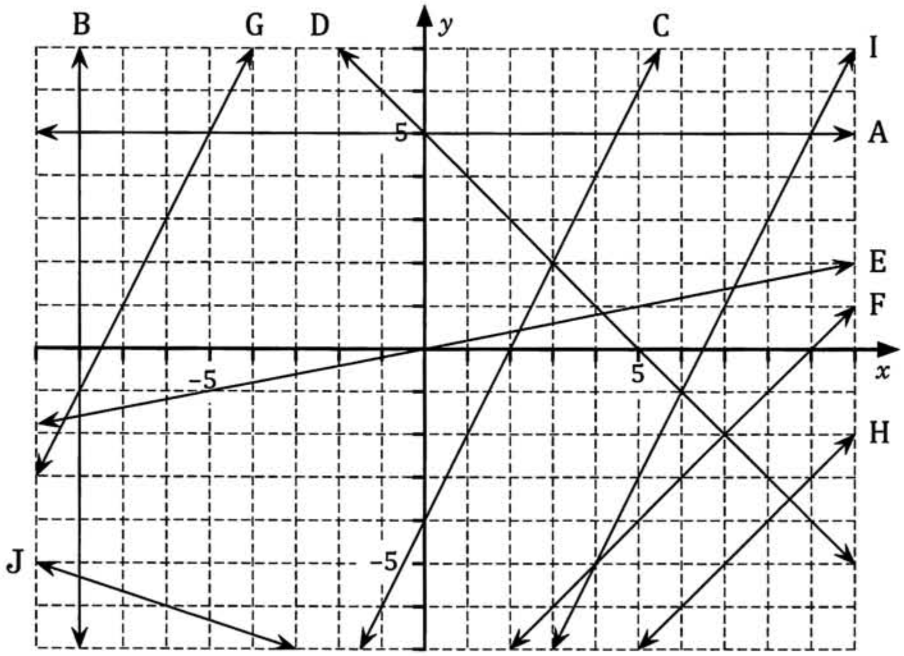
Source of data: The New Internationalist Magazine.

For the particular year and country involved:
(a) What percentage of the population lie below the poverty line?
(b) What is the "37·9%" in the above graph telling you?

10. Find the equation of the straight line with gradient 0·5, passing through (3, 4).
Each of the points F(9, *f*), G(−9, *g*), H(*h*, 9), I(*i*, 1·5) and J(3·8, *j*) lie on this line.
Determine the values of *f*, *g*, *h*, *i* and *j*.

11. A group of students sat an exam. The mean score for the boys was 56% and for the girls was 62%.
(a) If the group had the same number of girls as it had boys what would be the mean of the whole group?
(b) If the mean for the whole group was actually 59·8% were there more boys than girls in the group or were there more girls than boys?

12. Find the equations of the lines A to J shown below.



13. Scientists investigating levels of pollution in a particular stretch of river estimate that N, the number of fish that part of the river can support, depends on P, the number of tonnes of pollutant in that part of the river, according to the rule:

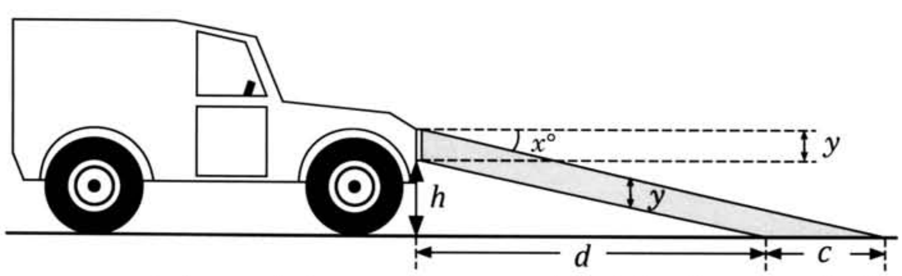
$$N \approx 60000 - 2100\,P$$

(a) How many fish does this rule suggest this stretch of the river can support if $P = 5$.

(b) If the level of pollution reaches 18 tonnes what number of fish could this stretch of the river support according to the above rule.

(c) If fish numbers are not to drop below 45 000 what does the formula suggest the pollution level must not exceed?

14. The diagram below shows a headlamp beam adjusted down to avoid dazzling oncoming drivers.



The light beam is angled at $x°$ below the horizontal and the distances h, d, c and y are as shown in the diagram. Assuming that the beam does not spread out at all and that the ground is horizontal, find d and c given that:

$$x = 4,\ h = 80 \text{ cm and } y = 20 \text{ cm}.$$